125. $\dfrac{-145.518}{18.42}$

126. $\dfrac{-297.078}{22.17}$

127. $\dfrac{12}{7} - \dfrac{17}{14} - \dfrac{48}{21}$

128. $\dfrac{9}{4} - \dfrac{21}{6} - \dfrac{11}{8}$

129. $54.2 - 18.78 - (-2.5) + 20.47$

130. $90.3 - 100.9 - (-34.26) + 32.95$

131. $(400)(-25.8)(0.003)$

132. $(500)(-12.4)(-0.02)$

133. $-\dfrac{11}{12} - \left(-\dfrac{1}{6}\right) + \dfrac{7}{8}$

134. $\dfrac{8}{15} - \left(-\dfrac{7}{9}\right) + \dfrac{2}{3}$

Applying the Concepts

135. Watching TV If Rachel spends $\frac{1}{8}$ of her life watching TV, how many hours of TV does she watch in one week?

136. Halloween Candy Henry decided to make $\frac{2}{3}$-oz bags of candy for treats at Halloween. If he bought 16 oz of candy, how many bags will he have to give away?

137. Biology Class Susan's biology class begins with 36 students. If $\frac{2}{3}$ will finish the course and $\frac{3}{4}$ of those get a passing grade, how many students will pass Susan's biology class this term?

138. Pizza Time Joyce and Ramie bought a pizza. Joyce ate $\frac{2}{5}$ of the pizza and Ramie ate $\frac{1}{9}$ of what was left. What fraction of the pizza remains uneaten?

139. Hourly Pay Last week, Jonathon received a paycheck for $442.80. The withholding for federal, state, and FICA (social security and Medicare) taxes was $97.20. If Jonathon worked 30 hours last week, what is his hourly pay rate?

140. Average Revenue Aqsa runs an online store selling beauty products. Last year, the total revenue of the company was $29,409.12. What was the average revenue per month?

141. Stock Prices The price per share of Intel stock has been up and down lately. On Monday it rose 2.75; on Tuesday it rose 0.87; on Wednesday it dropped 1.12; on Thursday it rose 0.52; and on Friday it fell 0.62. What was the net change in Intel's stock price per share for the week?

142. Bank Balance Henry started the month with $43.68 in his checking account. During the month the following transactions occurred: He deposited his paycheck of $929.30; and he wrote checks for rent $650, phone $33.49, credit card $229.50, cable service $75.50, and groceries $159.30. How much does he have in his account now?

Extending the Concepts

Problems 143–146 use the following definition.

If P and Q are two points on a real number line with coordinates a and b, respectively, then the **distance between P and Q**, denoted by $d(P, Q)$, is

$$d(P, Q) = |b - a|$$

143. Find the distance between the points P and Q on the real number line if $P = -9.7$ and $Q = 3.5$.

144. Find the distance between the points P and Q on the real number line if $P = -12.5$ and $Q = 2.6$.

145. Find the distance between the points P and Q on the real number line if $P = -\frac{13}{3}$ and $Q = \frac{7}{5}$.

146. Find the distance between the points P and Q on the real number line if $P = -\frac{5}{6}$ and $Q = 4$.

Explaining the Concepts

147. We know that 6 divided by 2 is 3. Explain why 6 divided by $\frac{1}{2}$ is 12.

148. Use a figure like Figure 17 on page 39 to explain why $\frac{1}{6} + \frac{2}{6} = \frac{1}{2}$.

Putting the Concepts Together (Sections 1.2–1.5)

We designed these problems so that you can review Sections 1.2–1.5 and show your mastery of the concepts. Take time to work these problems before proceeding with the next section. The answers to these problems are located at the back of the text on page AN-2.

1. Write $\frac{7}{8}$ and $\frac{9}{20}$ as equivalent fractions with the least common denominator.
2. Write $\frac{21}{63}$ as a fraction in lowest terms.
3. Convert $\frac{2}{7}$ to a decimal.
4. Write 0.375 as a fraction in lowest terms.
5. Write 12.3% as a decimal.
6. Write 0.0625 as a percent.
7. Use the set $\left\{-12, -\frac{14}{7}, -1.25, 0, \sqrt{2}, 3, 11.2\right\}$ to list all of the elements that are:
   (a) integers
   (b) rational numbers
   (c) irrational numbers
   (d) real numbers
8. Replace the ? with the correct symbol $>, <, =$: $\frac{1}{8}$? 0.5

In Problems 9–30, perform the indicated operation and write in lowest terms.

9. $17 + (-28)$
10. $-23 + (-42)$
11. $18 - 45$
12. $3 - (-24)$
13. $-18 - (-12.5)$
14. $(-5)(2)$
15. $25(-4)$
16. $(-8)(-9)$
17. $\frac{-35}{7}$
18. $\frac{-32}{-2}$
19. $27 \div -3$
20. $-\frac{4}{5} - \frac{11}{5}$
21. $7 - \frac{4}{5}$
22. $\frac{7}{12} + \frac{5}{18}$
23. $-\frac{5}{12} - \frac{1}{18}$
24. $\frac{6}{25} \cdot 15 \cdot \frac{1}{2}$
25. $\frac{2}{7} \div (-8)$
26. $\frac{0}{-8}$
27. $3.56 - (-7.2)$
28. $18.946 - 11.3$
29. $62.488 \div 42.8$
30. $(7.94)(2.8)$

1.6 Properties of Real Numbers

Objectives

1. Use the Identity Properties of Addition and Multiplication
2. Use the Commutative Properties of Addition and Multiplication
3. Use the Associative Properties of Addition and Multiplication
4. Understand the Multiplication and Division Properties of 0

Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

R1. Find the sum: $12 + 3 + (-12)$ [Section 1.4, pp. 28–30]

R2. Find the product: $\frac{3}{4} \cdot 11 \cdot \frac{4}{3}$ [Section 1.5, pp. 37–38]

This section presents properties of real numbers. A property in mathematics is a rule that is always true. These properties will be used throughout this text and in future math courses, so it is extremely important that you understand these properties and know how to use them.

1 Use the Identity Properties of Addition and Multiplication

The real number 0 is the only number that when added to any real number a results in the same real number a.

Ready?...Answers **R1.** 3 **R2.** 11

In Words
The word "identity" comes from a Latin word that means "the same" or "alike." The Identity Property of Addition means adding zero to a real number keeps the number the same.

Identity Property of Addition

For any real number a,

$$0 + a = a + 0 = a$$

That is, the sum of any number and 0 is that number. We call 0 the **additive identity.**

Recall that $3 \cdot 5$ is equivalent to adding 5 three times, so $3 \cdot 5 = 5 + 5 + 5$. Therefore, $1 \cdot 5$ means to add 5 once, so $1 \cdot 5 = 5$. This property about the real number 1 is true in general.

Identity Property of Multiplication

For any real number a,

$$a \cdot 1 = 1 \cdot a = a$$

That is, the product of any number and 1 is that number. We call 1 the **multiplicative identity.**

The multiplicative identity lets us create expressions that are equivalent to other expressions. For example, the expressions $\frac{4}{5}$ and $\frac{4}{5} \cdot \frac{3}{3}$ are equivalent because $\frac{3}{3} = 1$.

Conversion

One use of the multiplicative identity is *conversion.* **Conversion** is changing the units of measure (such as inches or pounds). For example, we might change a length from inches to feet or a weight from pounds to ounces.

EXAMPLE 1 Converting from Inches to Feet

Janice measures her family room and finds that its length is 184 inches. How many feet long is Janice's family room? (*Note*: 12 inches = 1 foot)

Solution

When doing conversions, make sure the units of measure you are trying to remove get divided out and the new units of measure remain. In this problem, we want the inches to divide out and feet to remain. Because 12 inches equals 1 foot, multiplying 184 inches by $\frac{1 \text{ foot}}{12 \text{ inches}}$ is the same as multiplying by 1.

Multiplying by 1

$$184 \text{ inches} = 184 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$$

$$= \frac{184}{12} \text{ feet}$$

$184 = 2 \cdot 2 \cdot 2 \cdot 23$; $12 = 2 \cdot 2 \cdot 3$; Divide out common factors:

$$= \frac{2 \cdot 2 \cdot 2 \cdot 23}{2 \cdot 2 \cdot 3} \text{ feet}$$

$$= \frac{46}{3} \text{ feet}$$

Work Smart

$$\begin{array}{r} 15 \\ 3\overline{)46} \\ -3 \\ \hline 16 \\ -15 \\ \hline 1 \end{array}$$

So 184 inches equals $\frac{46}{3}$ feet. Because 46 divided by 3 is 15 with a remainder of 1 (Why? See the Work Smart), $\frac{46}{3}$ feet is equivalent to $15\frac{1}{3}$ feet. Since $\frac{1}{3}$ foot $\cdot \frac{12 \text{ inches}}{1 \text{ foot}} = 4$ inches, we have that $\frac{46}{3}$ feet is equivalent to 15 feet, 4 inches.

Quick ✓

1. Because $a \cdot 1 = 1 \cdot a = a$ for any real number a, we call 1 the ________________.

In Problems 2–4, convert each measurement to the indicated unit of measurement.

2. 96 inches = ? feet [1 foot = 12 inches]
3. 500 minutes = ? hours [60 minutes = 1 hour]
4. 88 ounces = ? pounds [16 ounces = 1 pound]

2 Use the Commutative Properties of Addition and Multiplication

EXAMPLE 2 Illustrating the Commutative Properties

(a) $4 + 7 = 11$ and
$7 + 4 = 11$ so
$4 + 7 = 7 + 4$

(b) $3 \cdot 8 = 24$ and
$8 \cdot 3 = 24$ so
$3 \cdot 8 = 8 \cdot 3$

The results of Example 2 are true in general.

Commutative Properties of Addition and Multiplication

If a and b are real numbers, then

$$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a$$

In Words
The Commutative Property of real numbers states that the order in which we add or multiply real numbers does not affect the final result.

We add real numbers from left to right. We multiply real numbers from left to right. The **Commutative Property** allows us to write $3 + 5$ as $5 + 3$ and write $3 \cdot 5$ as $5 \cdot 3$ without affecting the value of the expression. Why is this important? Rearranging addition or multiplication problems makes some expressions easier to evaluate.

EXAMPLE 3 Using the Commutative Property of Addition

Evaluate the expression: $18 + 3 + (-18)$

Solution

$$\begin{aligned} 18 + 3 + (-18) &= 18 + (-18) + 3 \\ &= 0 + 3 \\ &= 3 \end{aligned}$$

If we add from left to right, we get $18 + 3 + (-18) = 21 + (-18) = 3$. Rearranging the numbers made the problem easier!

Does subtraction obey the Commutative Property? In other words, does $3 - 14 = 14 - 3$? Because $3 - 14 = -11$, but $14 - 3 = 11$, we see that **subtraction is not commutative.**

Quick ✓

5. The Commutative Property of Addition states that for any real numbers a and b, $a + b =$ _ + _.

6. The Commutative Property of Multiplication states that for any real numbers a and b, _ · _ = _ · _.

In Problems 7–9, use the Commutative Property of Addition and the Additive Inverse Property to find the sum of the real numbers.

7. $(-8) + 22 + 8$

8. $\frac{8}{15} + \frac{3}{20} + \left(-\frac{8}{15}\right)$

9. $2.1 + 11.98 + (-2.1)$

EXAMPLE 4 Using the Commutative Property of Multiplication

Find each product.

(a) $-27 \cdot 7 \cdot \left(-\frac{2}{9}\right)$

(b) $100 \cdot 307.5 \cdot 0.01$

Solution

(a)
$$\begin{aligned} -27 \cdot 7 \cdot \left(-\frac{2}{9}\right) &= -27 \cdot \left(-\frac{2}{9}\right) \cdot 7 \\ &= -\overset{3}{\cancel{27}} \cdot \left(-\frac{2}{\underset{1}{\cancel{9}}}\right) \cdot 7 \\ &= -3 \cdot (-2) \cdot 7 \\ &= 6 \cdot 7 \\ &= 42 \end{aligned}$$

(b)
$$\begin{aligned} 100 \cdot 307.5 \cdot 0.01 &= 100 \cdot 0.01 \cdot 307.5 \\ &= 1 \cdot 307.5 \\ &= 307.5 \end{aligned}$$

Quick ✓

In Problems 10–12, find the product of the real numbers.

10. $\left(-\frac{4}{3}\right) \cdot (-13) \cdot \left(-\frac{3}{4}\right)$

11. $\frac{5}{22} \cdot \frac{18}{331} \cdot \left(-\frac{44}{5}\right)$

12. $100{,}000 \cdot 349 \cdot 0.00001$

Work Smart

Neither division nor subtraction is commutative.

Does division obey the Commutative Property? That is, does $a \div b = b \div a$? For example, does $8 \div 2 = 2 \div 8$? Because $8 \div 2 = 4$, but $2 \div 8 = \frac{2}{8} = \frac{1}{4}$, we conclude that **division is not commutative.**

3 Use the Associative Properties of Addition and Multiplication

Sometimes **grouping symbols** such as parentheses ( ), brackets [ ], or braces { } are used to indicate that the operation within the grouping symbols is to be performed first. For example, $5 \cdot (8 + 3)$ means that we should first add 8 and 3 and then multiply this sum by 5.

Earlier we mentioned that addition is performed from left to right. We also stated that multiplication is performed from left to right. But does the order in which we add (or multiply) three or more numbers matter? Let's see.

EXAMPLE 5 Illustrating the Associative Properties

(a) $2 + (8 + 6) = 2 + 14 = 16$ and $(2 + 8) + 6 = 10 + 6 = 16$

so

$$2 + (8 + 6) = (2 + 8) + 6$$

(b) $-4 \cdot (9 \cdot 2) = -4 \cdot (18) = -72$ and $(-4 \cdot 9) \cdot 2 = -36 \cdot 2 = -72$

so

$$-4 \cdot (9 \cdot 2) = (-4 \cdot 9) \cdot 2$$

Example 5 illustrates the **Associative Properties of Addition and Multiplication.**

Associative Properties of Addition and Multiplication

If a, b, and c are real numbers, then

$$a + (b + c) = (a + b) + c = a + b + c$$
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c = a \cdot b \cdot c$$

EXAMPLE 6 Using the Associative Property of Addition

Use the Associative Property of Addition to evaluate $23 + 453 + (-453)$.

Solution

Because 453 and -453 are additive inverses, we use the Associative Property of Addition to insert parentheses around these numbers.

$$\begin{aligned} 23 + 453 + (-453) &= 23 + (453 + (-453)) \\ &= 23 + 0 \\ &= 23 \end{aligned}$$

EXAMPLE 7 Using the Associative Property of Multiplication

Use the Associative Property of Multiplication to evaluate $-\frac{3}{11} \cdot \frac{9}{4} \cdot \frac{8}{3}$.

Solution

Because the second and third factors have common factors that can be divided out, use the Associative Property of Multiplication to insert parentheses around $\frac{9}{4} \cdot \frac{8}{3}$ and perform this operation first.

$$\begin{aligned} -\frac{3}{11} \cdot \frac{9}{4} \cdot \frac{8}{3} &= -\frac{3}{11} \cdot \left(\frac{9}{4} \cdot \frac{8}{3}\right) \\ &= -\frac{3}{11} \cdot \left(\frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}}\right) \\ &= -\frac{3}{11} \cdot (3 \cdot 2) \\ &= -\frac{3}{11} \cdot 6 \\ &= -\frac{18}{11} \end{aligned}$$

Quick ✓

In Problems 13–16, use an Associative Property to evaluate each expression.

13. $14 + 101 + (-101)$

14. $14 \cdot \frac{1}{5} \cdot 5$

15. $-34.2 + 12.6 + (-2.6)$

16. $\frac{19}{2} \cdot \frac{4}{38} \cdot \frac{50}{13}$

4 Understand the Multiplication and Division Properties of 0

Now let's look at multiplication by zero.

Multiplication Property of Zero

For any real number a, the product of a and 0 is always 0; that is,

$$a \cdot 0 = 0 \cdot a = 0$$

We now introduce some division properties of zero.

Division Properties of Zero

For any nonzero real number a,

1. The quotient of 0 and a is 0. That is, $\frac{0}{a} = 0$.
2. The quotient of a and 0 is **undefined**. That is, $\frac{a}{0}$ is undefined.

Work Smart

Division by zero is not allowed. That is, 0 cannot be used as a divisor.

Why are these statements true? When we divide, we can check the quotient by multiplication. For example, $\frac{12}{4} = 3$ because $4 \cdot 3 = 12$. In the same way, $\frac{0}{4} = 0$ because $4 \cdot 0 = 0$. But what is the value of $\frac{12}{0}$? To determine this quotient, we should be able to determine a real number such that $0 \cdot \square = 12$. But since the product of 0 and any real number is 0, there is no value for $\square$.

EXAMPLE 8 Using Zero as a Divisor and a Dividend

Find the quotient:

(a) $\frac{23}{0}$ **(b)** $\frac{0}{17}$

Solution

(a) $\frac{23}{0}$ is undefined because 0 is the divisor.

(b) $\frac{0}{17} = 0$ because 0 is the dividend.

Quick ✓

In Problems 17 and 18, tell whether the quotient is zero or undefined.

17. $\frac{0}{22}$

18. $\frac{-11}{0}$

We now summarize the properties of addition, multiplication, and division.

Work Smart

The Commutative Property changes **order** and the Associative Property changes **grouping**.

Summary Properties of Addition

Identity Property of Addition For any real number a, $0 + a = a + 0 = a$.

Commutative Property of Addition If a and b are real numbers, then $a + b = b + a$.

Additive Inverse Property For any real number a, $a + (-a) = -a + a = 0$.

Associative Property of Addition If a, b, and c are real numbers, then $a + (b + c) = (a + b) + c$.

Summary Properties of Multiplication and Division

Identity Property of Multiplication $a \cdot 1 = 1 \cdot a = a$ for any real number a.

Commutative Property of Multiplication If a and b are real numbers, then $a \cdot b = b \cdot a$.

Multiplicative Inverse Property $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$ provided that $a \neq 0$.

Associative Property of Multiplication If a, b, and c are real numbers, then $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

Multiplication Property of Zero For any real number a, $a \cdot 0 = 0 \cdot a = 0$.

Division Properties of Zero For any nonzero number a, $\frac{0}{a} = 0$ and $\frac{a}{0}$ is undefined.

Work Smart: Study Skills

Selected problems in the exercise sets are identified by a green color. For extra help, worked solutions to these problems are in MyMathLab.

1.6 Exercises

MyMathLab® MathXL® PRACTICE

Exercise numbers in green have complete video solutions in MyMathLab.

Problems **1–18** *are the* Quick ✓s *that follow the* EXAMPLES.

Building Skills

In Problems 19–28, convert each measurement to the indicated unit of measurement. See Objective 1. Use the following conversions:

1 foot = 12 inches	3 feet = 1 yard
1 gallon = 4 quarts	100 centimeters = 1 meter
16 ounces = 1 pound	

19. 13 feet to inches

20. 13 yards to feet

21. 4500 centimeters to meters

22. 5900 centimeters to meters

23. 42 quarts to gallons

24. 58 quarts to gallons

25. 180 ounces to pounds

26. 120 ounces to pounds

27. 16,200 seconds to hours

28. 22,500 seconds to hours

In Problems 29–44, state the property of real numbers that is being illustrated. See Objectives 2, 3, and 4.

29. $16 + (-16) = 0$

30. $4 \cdot 63 \cdot \frac{1}{4} = 4 \cdot \frac{1}{4} \cdot 63$

31. $\frac{3}{4}$ is equivalent to $\frac{3}{4} \cdot \frac{5}{5}$

32. $4 + 5 + (-4)$ is equivalent to $4 + (-4) + 5$

33. $12 \cdot \frac{1}{12} = 1$

34. $-236 + 236 = 0$

35. $34.2 + (-34.2) = 0$

36. $(4 \cdot 5) \cdot 7 = 4 \cdot (5 \cdot 7)$

37. $\frac{0}{17}$

38. $\frac{-8}{0}$

39. $\frac{2}{3} \cdot \left(-\frac{12}{43}\right) \cdot \frac{3}{2} = \frac{2}{3} \cdot \frac{3}{2} \cdot \left(-\frac{12}{43}\right)$

40. $\frac{5}{12} \cdot \frac{12}{5} = 1$

41. $5.23 + 4.98 + (-5.23) = 5.23 + (-5.23) + 4.98$

42. $16.4 \cdot 0 = 0$

43. $\frac{21}{0}$

44. $\frac{0}{106}$

Mixed Practice

In Problems 45–64, evaluate each expression by using the properties of real numbers.

45. $54 + 29 + (-54)$

46. $46 + 59 + (-46)$

47. $\frac{9}{5} \cdot \frac{5}{9} \cdot 18$

48. $\frac{4}{9} \cdot \frac{9}{4} \cdot 28$

49. $-25 \cdot 13 \cdot \frac{1}{5}$

50. $36 \cdot (-12) \cdot \frac{1}{6}$

51. $347 + 456 + (-456)$

52. $593 + 306 + (-306)$

53. $\frac{9}{2} \cdot \left(-\frac{10}{3}\right) \cdot 6$

54. $\frac{13}{2} \cdot \frac{8}{39} \cdot \frac{39}{4}$

55. $\frac{7}{0}$

56. $\frac{0}{100}$

57. $100(-34)(0.01)$

58. $4000(0.5)(0.001)$

59. $569.003 \cdot 0$

60. $104 \cdot \frac{1}{104}$

61. $\frac{45}{3902} + \left(-\frac{45}{3902}\right)$

62. $30 \cdot \frac{4}{4}$

63. $-\frac{5}{44} \cdot \frac{80}{3} \cdot \frac{11}{5}$

64. $\frac{7}{48} \cdot \left(-\frac{21}{4}\right) \cdot \frac{12}{7}$

Applying the Concepts

65. **Balancing the Checkbook** Alberto's checking account balance at the start of the month was \$321.03. During the month, he wrote checks for \$32.84, \$85.03, and \$120.56. He also deposited a check for \$120.56. What is Alberto's balance at the end of the month?

66. **Stock Price** Before the opening bell on Monday, a certain stock was priced at \$32.04. On Monday the stock was up \$0.54, on Tuesday it was down \$0.32, and on Wednesday it was down \$0.54. What was the closing price of the stock on Wednesday?

Extending the Concepts

In Problems 67–70, insert parentheses to make the statement true.

67. $-3 - 4 - 10 = 3$

68. $-6 - 4 + 10 = -20$

69. $-15 + 10 - 4 - 8 = -1$

70. $25 - 6 - 10 - 1 = 28$

71. Convert 30 miles per hour to feet per second. (*Note*: 1 mile = 5280 feet)

72. Convert 40 miles per hour to feet per second. (*Note*: 1 mile = 5280 feet)

Explaining the Concepts

73. In your own words, explain why 0 does not have a multiplicative inverse.

74. Why does $2(4 \cdot 5)$ not equal $(2 \cdot 4) \cdot (2 \cdot 5)$?

75. Why does $\frac{0}{4} = 0$? Why is $\frac{4}{0}$ undefined?

76. How is the Identity Property of Addition related to the Additive Inverse Property?

77. How is the Identity Property of Multiplication related to the Multiplicative Inverse Property?

78. Let a be a real number such that $a > 0$, and let b represent a real number such that $b < 0$. Indicate whether each of the following statements are true or false, and explain your decision using actual values for a and b.
    (a) $a > -a$
    (b) $b > -b$
    (c) $a + (-a) = 0$
    (d) $-b + b = 0$
    (e) $a - (-b) > 0$
    (f) $ab < 0$
    (g) $-ab > 0$
    (h) $\frac{-a}{-b} > 0$

1.7 Exponents and the Order of Operations

Objectives

1. Evaluate Exponential Expressions
2. Apply the Rules for Order of Operations

Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

R1. Find the sum: $9 + (-19)$ [Section 1.4, pp. 28–30]

R2. Find the difference: $28 - (-7)$ [Section 1.4, pp. 31–32]

R3. Find the product: $-7 \cdot \frac{8}{3} \cdot 36$ [Section 1.5, pp. 37–38]

R4. Find the quotient: $\frac{100}{-15}$ [Section 1.4, pp. 34–35]

1 Evaluate Exponential Expressions

If we wanted to multiply 2 eight times, we would write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. That's a lot of writing! To reduce the amount of writing needed to show repeated multiplication, we use **exponential notation**, where we write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^8. In 2^8, 2 is called the **base** and 8 is called the **exponent**.

Exponential Notation

If n is a natural number and a is a real number, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$

where a is the **base** and n is the **exponent** or **power.** The exponent tells the number of times the base is used as a factor.

An expression written in the form a^n is said to be in **exponential form.** The expression 6^2 is read "six squared," 8^3 is read "eight cubed," and the expression 11^4 is read "eleven to the fourth power." In general, we read a^n as "a to the nth power."

EXAMPLE 1 Writing a Numerical Expression in Exponential Form

Write each expression in exponential form.

(a) $5 \cdot 5 \cdot 5$ **(b)** $(-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4)$

Solution

(a) The expression $5 \cdot 5 \cdot 5$ contains three factors of 5, so $5 \cdot 5 \cdot 5 = 5^3$.

(b) The expression $(-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4)$ contains six factors of -4, so $(-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) = (-4)^6$.

Quick ✓

1. In the expression 3^6, 3 is the ____ and 6 is the _______ or _____.

In Problems 2 and 3, write each expression in exponential form.

2. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$
3. $(-7) \cdot (-7) \cdot (-7) \cdot (-7)$

Ready?...Answers **R1.** -10 **R2.** 35 **R3.** -672 **R4.** $-\frac{20}{3}$

To evaluate an exponential expression, write the expression in **expanded form.** For example, 2^8 in expanded form is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.